Differential Equation Worksheet

1. $\frac{dy}{dx} = \frac{xy}{x^2+4}$ $\quad f(-2\sqrt{3}) = -4$

$(x^2+4)\,dy = xy\,dx$

$\int \frac{1}{y}\,dy = \int \frac{x}{x^2+4}\,dx$

$\ln|y|$ $\quad u = x^2+4$ $\quad \int \frac{x}{u}\cdot\frac{du}{2x}$

$\frac{du}{dx} = 2x$ $\quad \frac{1}{2}\int \frac{1}{u}\,du$

$dx = \frac{du}{2x}$ $\quad \frac{1}{2}\ln|x^2+4| + C$

$\ln|y| = \frac{1}{2}\ln|x^2+4| + C$

$\ln|-4| = \frac{1}{2}\ln|(-2\sqrt{3})^2+4| + C$

$\ln 4 = \frac{1}{2}\ln|16| + C$

$\ln 4 = \ln 4 + C$

$C = 0$

$\ln|y| = \frac{1}{2}\ln(x^2+4)$

$y = (x^2+4)^{1/2}$

2. $\frac{dy}{dx} = \frac{x+3}{2y}$ $\quad f(0) = -3$

$\int 2y\,dx = \int x+3\,dx$

$y^2 = \frac{1}{2}x^2 + 3x + C$

$(-3)^2 = \frac{1}{2}(0)^2 + 3(0) + C$

$9 = C$

$y^2 = \frac{1}{2}x^2 + 3x + 9$

$y = \pm\sqrt{\frac{1}{2}x^2+3x+9}$

$y = -\sqrt{\frac{1}{2}x^2+3x+9}$

3. $\frac{dy}{dx} = x\sqrt{y}$ $\quad f(3) = 25$

$dy = xy^{1/2}\,dx$

$\int \frac{1}{y^{1/2}}\,dy = \int x\,dx$

$2y^{1/2} = \frac{1}{2}x^2 + C$

$2(25)^{1/2} = \frac{1}{2}(3)^2 + C$

$10 = \frac{9}{2} + C$

$\frac{11}{2} = C$

$2y^{1/2} = \frac{1}{2}x^2 + \frac{11}{2}$

$y^{1/2} = \frac{1}{4}x^2 + \frac{11}{4}$

$y = \left(\frac{1}{4}x^2 + \frac{11}{4}\right)^2$

4. $\frac{dy}{dx} = 2xy\sqrt{x^2+3}$ $\quad f(1) = e^7$

$dy = 2xy\sqrt{x^2+3}\,dx$

$\int \frac{1}{y}\,dy = \int 2x\sqrt{x^2+3}\,dx$

$\ln|y|$ $\quad u = x^2+3$ $\quad \int 2x \cdot u^{1/2} \cdot \frac{du}{2x}$

$\frac{du}{dx} = 2x$ $\quad \int u^{1/2}\,du$

$dx = \frac{du}{2x}$ $\quad \frac{2}{3}u^{3/2} + C$

$\frac{2}{3}(x^2+3)^{3/2} + C$

$\ln|y| = \frac{2}{3}(x^2+3)^{3/2} + C$

$\ln|e^7| = \frac{2}{3}(1)^2+3)^{3/2} + C$

$7 = \frac{16}{3} + C$

$\frac{5}{3} = C$

$\ln|y| = \frac{2}{3}(x^2+3)^{3/2} + \frac{5}{3}$

$y = \pm e^{\frac{2}{3}(x^2+3)^{3/2} + \frac{5}{3}}$

$y = e^{\frac{2}{3}(x^2+3)^{3/2} + \frac{5}{3}}$

5. $a(t) = 3t^2 - 12t + 1$ $\quad v(0) = 2$

$s(4) = -53$

$v(t) = \int 3t^2 - 12t + 1 \, dt$ $\quad s(0) = ?$

$v(t) = t^3 - 6t^2 + t + C$

$2 = (0)^3 - 6(0)^2 + (0) + C$

$2 = C$

$v(t) = t^3 - 6t^2 + t + 2$

$s(t) = \int t^3 - 6t^2 + t + 2 \, dt$

$s(t) = \frac{1}{4}t^4 - 2t^3 + \frac{1}{2}t^2 + 2t + C$

$-53 = \frac{1}{4}(4)^4 - 2(4) + \frac{1}{2}(4)^2 + 2(4) + C$

$-53 = 64 - 8 + 8 + 8 + C$

$-53 = 72 + C$

$-125 = C$

$s(t) = \frac{1}{4}t^4 - 2t^3 + \frac{1}{2}t^2 + 2t - 125$

$s(0) = -125$ ft.